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# Why does manipulation of social choices matter?

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## Abstract

I discuss why nonmanipulation of social choices is an important area of research.

**Keywords:** manipulation, social choice, strategy-proofness.

**JEL classification:** D71.

## 1 Introduction

In this article, I discuss reasons why we consider *nonmanipulability* to be an important property in the context of social choice. *Nonmanipulability* of a social choice rule (simply, a rule) has been one of the most important topics in the theory of social choice,<sup>1</sup> but its significance might not be clear to some researchers outside the field. As a researcher of *nonmanipulability*, I try to explain why I believe *nonmanipulability* to be an important property of rules.

In Section 2, I formally define the most famous formulation of *nonmanipulability*, i.e., *strategy-proofness*. In Section 3, I discuss the significance of *nonmanipulability* and recent researches.

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<sup>1</sup>See Barberà (2010) for a survey.

## 2 Basic notation and definitions

Let  $N = \{1, \dots, n\}$  be a finite set of *agents*, and  $X$  be a finite set of *alternatives*. For example, in the context of voting,  $N$  is a set of voters and  $X$  is a set of candidates. Let  $\mathcal{L}$  be the set of all *linear orders* on  $X$ . Elements of  $\mathcal{L}$  represent *preferences*. For each  $i \in N$ , each  $R_i \in \mathcal{L}$ , and each  $x, y \in X$ ,  $xR_iy$  means that for agent  $i$ ,  $x$  is at least as good as  $y$ .<sup>2</sup> Let  $\mathcal{L}^N$  denote the set of all  $n$ -tuples  $\mathbf{R} = (R_1, \dots, R_n)$ , where  $R_i \in \mathcal{L}$  for each  $i \in N$ . Elements of  $\mathcal{L}^N$  are *preference profiles*. A *rule* is a function from  $\mathcal{L}^N$  into  $X$ . Generic notation for a rule is  $f$ .

We say that agent  $i$  *manipulates* a social choice if

- (i) he reports a false preference relation, and
- (ii) as a result, a social choice changes from the one under the true preference relation.

Thus, generally, misrepresentation is not equivalent to manipulation. (Misrepresentation does not necessarily change the social choice.)

A rule is *nonmanipulable* if each agent never manipulates a social choice under some assumption on agents' behavior.

**Assumption** (throughout the paper): Given  $\mathbf{R}_{-i} = (R_1, \dots, R_{i-1}, R_{i+1}, \dots, R_n)$ , agent  $i$  whose true preference relation is  $R_i$  reports a false preference relation  $R'_i$  only if  $f(R'_i, \mathbf{R}_{-i}) \neq f(R_i, \mathbf{R}_{-i})$ .

Under this assumption, misrepresentation is equivalent to manipulation.

A representative notion of *nonmanipulability* is *strategy-proofness*.

A rule  $f$  is *strategy-proof* if for each  $\mathbf{R} \in \mathcal{L}^N$ , each  $i \in N$ , and each  $R'_i \in \mathcal{L}$ ,

$$f(\mathbf{R})R_i f(R'_i, \mathbf{R}_{-i}),$$

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<sup>2</sup>Each linear order can be considered as a ranking of the alternatives without any ties between distinct alternatives. Thus,  $xR_iy$  and  $yR_ix$  if and only if  $x = y$ .

where  $(R'_i, R_{-i})$  is the preference profile obtained by replacing  $R_i$  in  $R$  by  $R'_i$ .

In the above definition of *strategy-proofness*,  $R_i$  is interpreted as agent  $i$ 's *true* preference relation, and  $R'_i$  is interpreted as agent  $i$ 's *false* preference relation. Then, the relation  $f(R)R_i f(R'_i, R_{-i})$  says that reporting  $R'_i$  is not beneficial for agent  $i$  regardless of what the other agents report. (Remember that the choice of  $R$  was arbitrary.) It is reasonable to assume that the agents report their preferences sincerely when misrepresentation of preferences is not profitable. Thus, *strategy-proofness* is a property which makes the rule designer reasonably conclude that the rule is *nonmanipulable*.

It would be safe to say that the Gibbard-Satterthwaite theorem (Gibbard, 1973; Satterthwaite, 1975) is the starting point of the theory of manipulation. The theorem says that under mild conditions, *strategy-proofness* is achieved only by *dictatorship*. Because of its negative implication, the theorem is called an *impossibility theorem*.

### 3 Discussion

#### 3.1 Designer's objectives

I do not intend to claim that *nonmanipulability* should be a universally critical property of a rule. Whether *nonmanipulability* is a critical property of a rule or not depends on the objective of the rule designer. I consider the following two types of objectives.

First, consider the designer who wants each social choice to have some "nice" relation to the agents' *true* preferences. For him, a rule describes a desirable relationship between the agents' preferences and social choices. To achieve the rule, it is necessary to elicit true preferences from the agents.

On the other hand, if the designer's objective is to construct a rule which re-

sponds “nicely” to the reported preferences, then *nonmanipulability* is not an important property at all. For the designer, a rule describes a desirable relationship between the reported preferences and social choices. Thus, there is no need to elicit true preferences from the agents.

There is no theory according to which one of the two attitudes of the designer is superior to the other. Thus, whether *nonmanipulability* matters or not depends on the designer’s subjective opinion about what a social choice should be related to. Nevertheless, in the following, I argue that economists, or more broadly, social scientists tend to take the former viewpoint, and this is the reason why *manipulation* is a significant subject of research.

### 3.2 Need for true preferences

Many axioms of rules refer to agents’ preferences. As a representative example, consider *Pareto efficiency*. For each  $R \in \mathcal{L}^N$ ,  $x \in X$  is *Pareto efficient* (simply, *efficient*) if there is no  $y \in X \setminus \{x\}$  such that  $xR_i y$  for each  $i \in N$ . (Remember that  $R_i$  is a linear order. Because  $x$  and  $y$  are distinct,  $xR_i y$  means that  $x$  is preferred to  $y$ .) A rule  $f$  is *efficient* if for each  $R \in \mathcal{L}^N$ ,  $f(R)$  is *efficient*.<sup>3</sup> The following simple arguments show that we usually interpret  $R_i$  as agent  $i$ ’s true preferences when we discuss *efficiency* of alternatives.

Let each agent  $i$  choose one linear order  $R_i$  randomly from  $\mathcal{L}$ . (It is possible that  $R_i$  happens to be the agent  $i$ ’s true preference relation, but generally, it is not necessarily the true one.) Let  $R$  be the profile of them. Each preference relation in  $R$  is nothing to do with agents’ true preferences. Then, most researchers would agree that finding an *efficient* alternative with respect to  $R$  is just a mathematical exercise, and that as economists, we do not put an importance on such exercises.

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<sup>3</sup>This notion is often called *weak Pareto efficiency*. Because we do not consider weak orders, there is no difference between *Pareto efficiency* and *weak Pareto efficiency* in this article.

We are interested in realizing an *efficient* alternative with respect to a profile  $R$  only if  $R_1, \dots, R_n$  are agents' true preferences. Therefore, we need to elicit information on true preferences from the agents.

## 4 Present situation and recent researches

*Strategy-proofness* has been a central axiom for *nonmanipulability*. However, in many situations, we have impossibility results with *strategy-proofness*, i.e., every acceptable or plausible rule violates *strategy-proofness*. Thus, we cannot recommend rules based on *strategy-proofness*. Although impossibility theorems should be the starting point for further researches, unfortunately, we do not have satisfactory solutions to many of them. Manipulation is a serious problem in social choice, and we cannot deviate from making social choice. Therefore, we have to find a class of acceptable rules which are less susceptible to manipulation.

There are several lines of researches. Maus, Peters, and Storcken (2007a,b,c,d) count the number of the profiles at which profitable misrepresentation occur. Barbie, Puppe, and Tasnádi (2006) and Sanver (2009) find domains under which fixed rules are *strategy-proof*. Campbell and Kelly (2009, 2010) consider gains and losses from manipulation. Sato (2011a,b) considers the reluctance to make large lies. However, it is clear that there is still much to be done.

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